

Full-Vectorial Finite Element Beam Propagation Method with Perfectly Matched Layers for Anisotropic Optical Waveguides

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Abstract—Perfectly matched layer (PML) boundary conditions are incorporated into the full-vectorial beam propagation method (BPM) based on a finite element scheme for the three-dimensional (3-D) anisotropic optical waveguide analysis. In the present approach, edge elements based on linear-tangential and quadratic-normal vector basis functions are used for the transverse field components. To show the validity and usefulness of this approach, numerical examples are shown for Gaussian beam propagation in proton-exchanged LiNbO₃ optical waveguides. Numerical accuracy of the present PML boundary condition is investigated in detail by comparing the results with those of the conventional absorbing boundary condition (ABC).

Index Terms—Anisotropic optical waveguide, beam propagation method (BPM), finite element method, full-wave analysis, perfectly matched layer (PML).

I. INTRODUCTION

THE beam propagation method (BPM) is at present the most widely used for the study of light propagation in longitudinally varying waveguides. Nowadays, there are a great number of versions of BPM [1]; BPM based on the fast Fourier transform (FFT-BPM), BPM based on the finite difference method (FD-BPM), and BPM based on the finite element method (FE-BPM) [2]–[5]. In particular, a recently developed finite element BPM is applicable to the most general (arbitrarily-shaped, inhomogeneous, dissipative, anisotropic, and nonlinear) optical waveguide problems. One of the key issues in implementing FE-BPM to study light propagation in a finite spatial domain is boundary conditions at the computational window (CW) edges. Berenger [6] has recently introduced the concept of a perfectly matched layer (PML) for reflectionless absorption of electromagnetic waves which can be employed as an alternative to the conventional absorbing boundary condition (ABC) [7], [8] or the transparent boundary condition (TBC) [9]. Unfortunately, since Berenger's PML technique involves a modification of Maxwell's equations based on the splitting of field components into two subcomponents, these non-Maxwellian equations do not have a desirable form for finite element (FE) formulations. More recently, new formulations of PML which do not involve the field splitting have been proposed by Pekel *et al.* [10] and Sacks *et al.* [11] and have been widely used for mesh truncation in

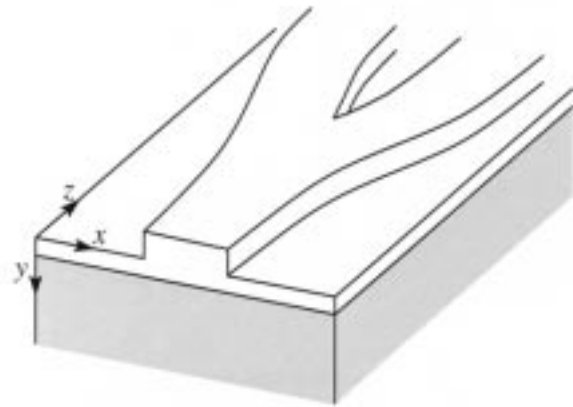


Fig. 1. Longitudinally varying 3-D optical waveguide.

FE-BPM [12], [13]. However, since these formulations are for isotropic waveguides, they cannot be applied to anisotropic optical waveguides. Although the TBC may be applicable to anisotropic optical waveguides, the formulation is limited to the approximate scalar analysis [14].

In this paper, the PML for anisotropic media [15] is, for the first time, incorporated into the full-vectorial FE-BPM (VFE-BPM) for the three-dimensional (3-D) anisotropic optical waveguide analysis. In the present formulation, edge elements based on linear-tangential and quadratic-normal vector basis functions [16]–[18] are used for the transverse field components. To show the validity and usefulness of this approach, numerical results are shown for Gaussian beam propagation in proton-exchanged LiNbO₃ optical waveguides. Numerical accuracy of the present PML boundary condition is investigated in detail by comparing the results with those of the conventional ABC.

II. BASIC EQUATIONS

We consider a longitudinally varying 3-D optical waveguide as shown in Fig. 1 and the transverse cross section of the optical waveguide surrounded by PML regions 1–8 with thicknesses d_j ($j = 1$ to 4) as shown in Fig. 2, where x and y are the transverse directions, z is the propagation direction, and the relative permittivity tensor is given arbitrarily as

$$[\epsilon_r] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}. \quad (1)$$

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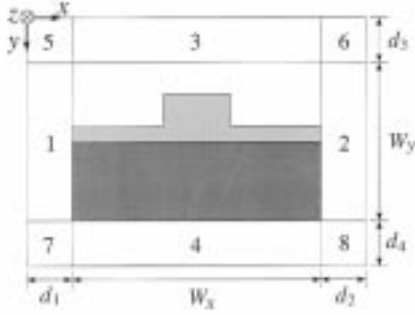


Fig. 2. Transverse cross section of 3-D optical waveguide surrounded by PMLs.

Using PML for anisotropic media [15], from Maxwell's equations the following vectorial wave equation is derived:

$$\nabla \times ([p]\nabla \times \Phi) - k_0^2[q]\Phi = 0 \quad (2)$$

where k_0 is the free-space wavenumber, Φ denotes either the electric field $\mathbf{E}$ or the magnetic field $\mathbf{H}$, and the matrices $[p]$ and $[q]$ are given by

$$\begin{aligned} [p] &= \begin{bmatrix} p_{xx} & p_{xy} & p_{xz} \\ p_{yx} & p_{yy} & p_{yz} \\ p_{zx} & p_{zy} & p_{zz} \end{bmatrix} \\ &= \begin{bmatrix} \frac{s_y s_z}{s_x} & 0 & 0 \\ 0 & \frac{s_z s_x}{s_y} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z} \end{bmatrix}^{-1} \\ [q] &= \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} \\ q_{yx} & q_{yy} & q_{yz} \\ q_{zx} & q_{zy} & q_{zz} \end{bmatrix} \\ &= \begin{bmatrix} \frac{s_y s_z}{s_x} \epsilon_{xx} & s_z \epsilon_{xy} & s_y \epsilon_{xz} \\ s_z \epsilon_{yx} & \frac{s_z s_x}{s_y} \epsilon_{yy} & s_x \epsilon_{yz} \\ s_y \epsilon_{zx} & s_x \epsilon_{zy} & \frac{s_x s_y}{s_z} \epsilon_{zz} \end{bmatrix} \end{aligned} \quad (3)$$

for $\Phi = \mathbf{E}$ and

$$\begin{aligned} [p] &= \begin{bmatrix} p_{xx} & p_{xy} & p_{xz} \\ p_{yx} & p_{yy} & p_{yz} \\ p_{zx} & p_{zy} & p_{zz} \end{bmatrix} \\ &= \begin{bmatrix} \frac{s_y s_z}{s_x} \epsilon_{xx} & s_z \epsilon_{xy} & s_y \epsilon_{xz} \\ s_z \epsilon_{yx} & \frac{s_z s_x}{s_y} \epsilon_{yy} & s_x \epsilon_{yz} \\ s_y \epsilon_{zx} & s_x \epsilon_{zy} & \frac{s_x s_y}{s_z} \epsilon_{zz} \end{bmatrix}^{-1} \\ [q] &= \begin{bmatrix} q_{xx} & q_{xy} & q_{xz} \\ q_{yx} & q_{yy} & q_{yz} \\ q_{zx} & q_{zy} & q_{zz} \end{bmatrix} = \begin{bmatrix} \frac{s_y s_z}{s_x} & 0 & 0 \\ 0 & \frac{s_z s_x}{s_y} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z} \end{bmatrix} \end{aligned} \quad (4)$$

with

$$\Phi = [\phi_x \quad \phi_y \quad \phi_z]^T \quad (11)$$

$$[A] = \begin{bmatrix} -p_{yy} & p_{yx} & 0 \\ p_{xy} & -p_{xx} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (12)$$

TABLE I
PML PARAMETERS

PML parameter	PML region							
	1	2	3	4	5	6	7	8
s_x	s_1	s_2	1	1	s_1	s_2	s_1	s_2
s_y	1	1	s_3	s_4	s_3	s_3	s_4	s_4

$s_z = 1$ in all the PML regions.

for $\Phi = \mathbf{H}$. The PML parameters s_x , s_y , and s_z are given in Table I, where the complex values of s_j ($j = 1$ to 4) are taken as

$$s_j = 1 - j\alpha_j. \quad (7)$$

Attenuation of the field Φ in PML regions can be controlled by choosing the appropriate values of α_j . Here, we assume parabolic profiles of s_j as

$$\alpha_j = \alpha_{j,\max} \left(\frac{\rho}{d_j} \right)^2 \quad (8)$$

where ρ is the distance from the beginning of PML and the subscript "max" denotes the maximum value.

Now, using an appropriate reference refractive index n_0 and slowly varying envelope approximation (SVEA), we assume the following form of the solution:

$$\begin{aligned} \Phi(x, y, z) &= \phi_x(x, y, z) \exp(-jk_0 n_0 z) \mathbf{i}_x \\ &+ \phi_y(x, y, z) \exp(-jk_0 n_0 z) \mathbf{i}_y \\ &+ \phi_z(x, y, z) \exp(-jk_0 n_0 z) \mathbf{i}_z. \end{aligned} \quad (9)$$

Substituting (9) into (2) and noting that the variation of p_{kl} along the z direction is sufficiently small, $\partial p_{kl} / \partial z \simeq 0$, we obtain the basic equation for the beam propagation analysis as

$$\begin{aligned} [A] \left(\frac{\partial^2}{\partial z^2} - 2jk_0 n_0 \frac{\partial}{\partial z} - k_0^2 n_0^2 \right) \Phi &+ [B_x] \left(\frac{\partial}{\partial z} - jk_0 n_0 \right) \frac{\partial \Phi}{\partial x} \\ &+ [B_y] \left(\frac{\partial}{\partial z} - jk_0 n_0 \right) \frac{\partial \Phi}{\partial y} \\ &+ \frac{\partial}{\partial x} \left\{ [C_x] \left(\frac{\partial}{\partial z} - jk_0 n_0 \right) \right. \\ &+ [D_x] \frac{\partial}{\partial x} + [E_x] \frac{\partial}{\partial y} \left. \right\} \Phi \\ &+ \frac{\partial}{\partial y} \left\{ [C_y] \left(\frac{\partial}{\partial z} - jk_0 n_0 \right) \right. \\ &+ [E_y] \frac{\partial}{\partial x} + [D_y] \frac{\partial}{\partial y} \left. \right\} \Phi + k_0^2 [q] \Phi = 0 \end{aligned} \quad (10)$$

$$[B_x] = \begin{bmatrix} 0 & -p_{yz} & p_{yy} \\ 0 & p_{xz} & -p_{xy} \\ 0 & 0 & 0 \end{bmatrix} \quad (13)$$

$$[B_y] = \begin{bmatrix} p_{yz} & 0 & -p_{yx} \\ -p_{xz} & 0 & p_{xx} \\ 0 & 0 & 0 \end{bmatrix} \quad (14)$$

$$[C_x] = \begin{bmatrix} 0 & 0 & 0 \\ -p_{zy} & p_{zx} & 0 \\ p_{yy} & -p_{yx} & 0 \end{bmatrix} \quad (15)$$

$$[C_y] = \begin{bmatrix} p_{zy} & -p_{zx} & 0 \\ 0 & 0 & 0 \\ -p_{xy} & p_{xx} & 0 \end{bmatrix} \quad (16)$$

$$[D_x] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -p_{zz} & p_{zy} \\ 0 & p_{yz} & -p_{yy} \end{bmatrix} \quad (17)$$

$$[D_y] = \begin{bmatrix} -p_{zz} & 0 & p_{zx} \\ 0 & 0 & 0 \\ p_{xz} & 0 & -p_{xx} \end{bmatrix} \quad (18)$$

$$[E_x] = \begin{bmatrix} 0 & 0 & 0 \\ p_{zz} & 0 & -p_{zx} \\ -p_{yz} & 0 & p_{yx} \end{bmatrix} \quad (19)$$

$$[E_y] = \begin{bmatrix} 0 & p_{zz} & -p_{zy} \\ 0 & 0 & 0 \\ 0 & -p_{xz} & p_{xy} \end{bmatrix} \quad (20)$$

where the superscript T denotes a transpose.

Assuming $\partial/\partial z \equiv 0$ and regarding n_0 as an effective refractive index, (10) is reduced to a basic equation for the guided-mode analysis of anisotropic optical waveguides.

III. FINITE ELEMENT DISCRETIZATION

Dividing the waveguide cross section into hybrid edge/nodal elements based on linear tangential and quadratic normal vector basis functions [16]–[18] as shown in Fig. 3, we expand the transverse field components ϕ_x , ϕ_y and the longitudinal field component ϕ_z in each element as

$$\phi = \begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} = \begin{bmatrix} \{U\}^T \{\phi_t\}_e \\ \{V\}^T \{\phi_t\}_e \\ j\{N\}^T \{\phi_z\}_e \end{bmatrix} \quad (21)$$

where

- $\{\phi_t\}_e$ and $\{\phi_z\}_e$ edge and nodal variables for each element, respectively;
- $\{U\}$ and $\{V\}$ shape function vectors for edge elements;
- $\{N\}$ shape function vector for nodal elements [18].

Substituting (21) into (10) and using the finite element technique to the transverse xy -plane, we obtain the following matrix equation:

$$[M] \frac{d^2 \{\phi\}}{dz^2} + ([L] - 2jk_0 n_0 [M]) \frac{d \{\phi\}}{dz} + ([K] - jk_0 n_0 [L] - k_0^2 n_0^2 [M]) \{\phi\} = \{0\} \quad (22)$$

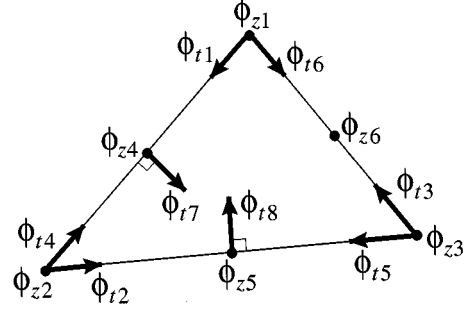


Fig. 3. Hybrid edge/nodal element based on linear tangential and quadratic normal vector basis functions.

with

$$\{\phi\} = \begin{bmatrix} \{\phi_t\} \\ \{\phi_z\} \end{bmatrix} \quad (23)$$

$$[K] = \begin{bmatrix} [K_{tt}] & j[K_{tz}] \\ -j[K_{zt}] & [K_{zz}] \end{bmatrix} \quad (24)$$

$$[L] = \begin{bmatrix} [L_{tt}] & j[L_{tz}] \\ j[L_{zt}] & [0] \end{bmatrix} \quad (25)$$

$$[M] = \begin{bmatrix} [M_{tt}] & [0] \\ [0] & [0] \end{bmatrix} \quad (26)$$

$$[K_{tt}] = \sum_e \iint_e \left[p_{zz} \frac{\partial \{U\}}{\partial y} \frac{\partial \{V\}^T}{\partial x} - p_{zz} \frac{\partial \{U\}}{\partial y} \frac{\partial \{U\}^T}{\partial y} \right. \\ \left. + k_0^2 q_{xx} \{U\} \{U\}^T + k_0^2 q_{xy} \{U\} \{V\}^T \right. \\ \left. - p_{zz} \frac{\partial \{V\}}{\partial x} \frac{\partial \{V\}^T}{\partial x} + p_{zz} \frac{\partial \{V\}}{\partial x} \frac{\partial \{U\}^T}{\partial y} \right. \\ \left. + k_0^2 q_{yx} \{V\} \{U\}^T \right. \\ \left. + k_0^2 q_{yy} \{V\} \{V\}^T \right] dx dy \quad (27)$$

$$[K_{tz}] = \sum_e \iint_e \left[p_{zx} \frac{\partial \{U\}}{\partial y} \frac{\partial \{N\}^T}{\partial y} - p_{zy} \frac{\partial \{U\}}{\partial y} \frac{\partial \{N\}^T}{\partial x} \right. \\ \left. - p_{zx} \frac{\partial \{V\}}{\partial x} \frac{\partial \{N\}^T}{\partial y} + p_{zy} \frac{\partial \{V\}}{\partial x} \frac{\partial \{N\}^T}{\partial x} \right. \\ \left. + k_0^2 q_{xz} \{U\} \{N\}^T \right. \\ \left. + k_0^2 q_{yz} \{V\} \{N\}^T \right] dx dy \quad (28)$$

$$[K_{zt}] = \sum_e \iint_e \left[p_{xz} \frac{\partial \{N\}}{\partial y} \frac{\partial \{U\}^T}{\partial y} - p_{yz} \frac{\partial \{N\}}{\partial x} \frac{\partial \{U\}^T}{\partial y} \right. \\ \left. - p_{xz} \frac{\partial \{N\}}{\partial y} \frac{\partial \{V\}^T}{\partial x} + p_{yz} \frac{\partial \{N\}}{\partial x} \frac{\partial \{V\}^T}{\partial x} \right. \\ \left. + k_0^2 q_{zx} \{N\} \{U\}^T \right. \\ \left. + k_0^2 q_{zy} \{N\} \{V\}^T \right] dx dy \quad (29)$$

$$[K_{zz}] = \sum_e \iint_e \left[p_{yx} \frac{\partial \{N\}}{\partial x} \frac{\partial \{N\}^T}{\partial y} - p_{yy} \frac{\partial \{N\}}{\partial x} \frac{\partial \{N\}^T}{\partial x} \right. \\ \left. - p_{yx} \frac{\partial \{N\}}{\partial y} \frac{\partial \{N\}^T}{\partial y} + p_{xy} \frac{\partial \{N\}}{\partial y} \frac{\partial \{N\}^T}{\partial x} \right. \\ \left. + k_0^2 q_{zz} \{N\} \{N\}^T \right] dx dy \quad (30)$$

$$[L_{tt}] = \sum_e \iint_e \left[p_{yz}\{U\} \frac{\partial\{V\}^T}{\partial x} - p_{yz}\{U\} \frac{\partial\{U\}^T}{\partial y} \right. \\ \left. - p_{zx} \frac{\partial\{U\}}{\partial y} \{V\}^T + p_{zy} \frac{\partial\{U\}}{\partial y} \{U\}^T \right. \\ \left. - p_{xz}\{V\} \frac{\partial\{V\}^T}{\partial x} + p_{xz}\{V\} \frac{\partial\{U\}^T}{\partial y} \right. \\ \left. + p_{zx} \frac{\partial\{V\}}{\partial x} \{V\}^T \right. \\ \left. - p_{zy} \frac{\partial\{V\}}{\partial x} \{U\}^T \right] dx dy \quad (31)$$

$$[L_{tz}] = \sum_e \iint_e \left[p_{yx}\{U\} \frac{\partial\{N\}^T}{\partial y} - p_{yy}\{U\} \frac{\partial\{N\}^T}{\partial x} \right. \\ \left. - p_{xx}\{V\} \frac{\partial\{N\}^T}{\partial y} \right. \\ \left. + p_{xy}\{V\} \frac{\partial\{N\}^T}{\partial x} \right] dx dy \quad (32)$$

$$[L_{zt}] = \sum_e \iint_e \left[p_{xy} \frac{\partial\{N\}}{\partial y} \{U\}^T - p_{yy} \frac{\partial\{N\}}{\partial x} \{U\}^T \right. \\ \left. - p_{xx} \frac{\partial\{N\}}{\partial y} \{V\}^T \right. \\ \left. + p_{yx} \frac{\partial\{N\}}{\partial x} \{V\}^T \right] dx dy \quad (33)$$

$$[M_{tt}] = \sum_e \iint_e \left[-p_{yx}\{U\}\{V\}^T + p_{yy}\{U\}\{U\}^T \right. \\ \left. + p_{xx}\{V\}\{V\}^T - p_{xy}\{V\}\{U\}^T \right] dx dy \quad (34)$$

where $\{0\}$ is a null vector, $[0]$ is a null matrix, and $\sum_e$ extends over all different elements. We can rewrite (22) formally as

$$[\tilde{M}] \frac{d\{X\}}{dz} + [\tilde{K}]\{X\} = \{0\} \quad (35)$$

with

$$\{X\} = \begin{bmatrix} \{\phi\} \\ \frac{d\{\phi_t\}}{dz} \end{bmatrix} \quad (36)$$

$$[\tilde{K}] = \begin{bmatrix} [K] - jk_0 n_0 [L] - k_0^2 n_0^2 [M] & [0] \\ [0] & -[M_{tt}] \end{bmatrix} \quad (37)$$

$$[\tilde{M}] = \begin{bmatrix} [L] - 2jk_0 n_0 [M] & [M_{tt}] \\ [M_{tt}] & [0] \end{bmatrix}. \quad (38)$$

For the diagonal relative permittivity tensor, $\varepsilon_{kl} = 0$ ($k \neq l$), using the longitudinal-field transformation [5]

$$\phi_z(x, y, z) \exp(-jk_0 n_0 z) \\ = j \frac{\partial}{\partial z} \{\phi'_z(x, y, z) \exp(-jk_0 n_0 z)\} \quad (39)$$

and the Padé approximation [2], [3], [12], [14], $\{X\}$, $[\tilde{K}]$, and $[\tilde{M}]$ are reduced to smaller sized ones as

$$\{X\} = \begin{bmatrix} \{\phi_t\} \\ \{\phi'_z\} \end{bmatrix} \quad (40)$$

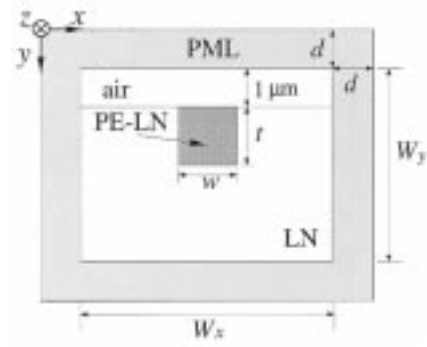


Fig. 4. Anisotropic optical waveguide surrounded by PMLs.

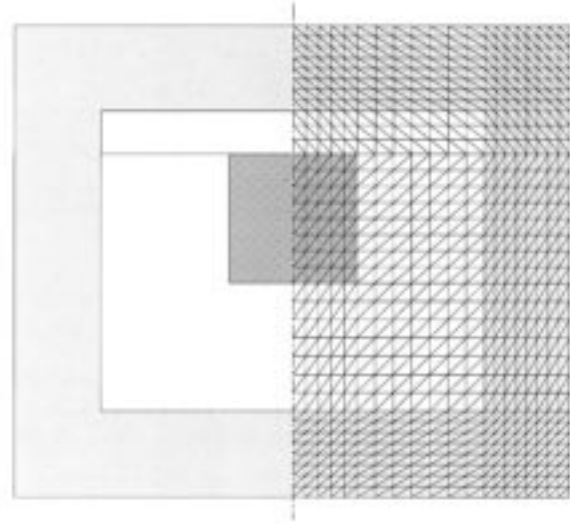


Fig. 5. Element division profile.

$$[\tilde{K}] = \begin{bmatrix} [K_{tt}] & [0] \\ [0] & [0] \end{bmatrix} - k_0^2 n_0^2 \begin{bmatrix} [M_{tt}] & -[L_{tz}] \\ -[L_{zt}] & -[K_{zz}] \end{bmatrix} \quad (41)$$

$$[\tilde{M}] = -2jk_0 n_0 \left(\begin{bmatrix} [M_{tt}] & -[L_{tz}] \\ -[L_{zt}] & -[K_{zz}] \end{bmatrix} + \frac{1}{4k_0^2 n_0^2} [\tilde{K}] \right). \quad (42)$$

IV. TWO-POINT RECURRENCE SCHEME

Applying the 2-point recurrence scheme for the propagation direction z to (35) yields

$$[A]_i \{X\}_{i+1} = [B]_i \{X\}_i \quad (43)$$

$$[A]_i = [\tilde{M}]_i + \theta \Delta z [\tilde{K}]_i \quad (44)$$

$$[B]_i = [\tilde{M}]_i - (1 - \theta) \Delta z [\tilde{K}]_i \quad (45)$$

where Δz is the propagation step size, θ is introduced to control the stability of the method, and the subscripts i and $(i + 1)$ denote the quantities related to the i th and $(i + 1)$ th propagation steps, respectively. Generally, the stability range for the propagation algorithm is $\theta \geq 0.5$, but unstable field evolution was observed with $\theta = 0.5$, Crank-Nicholson algorithm. So, $\theta = 1$,

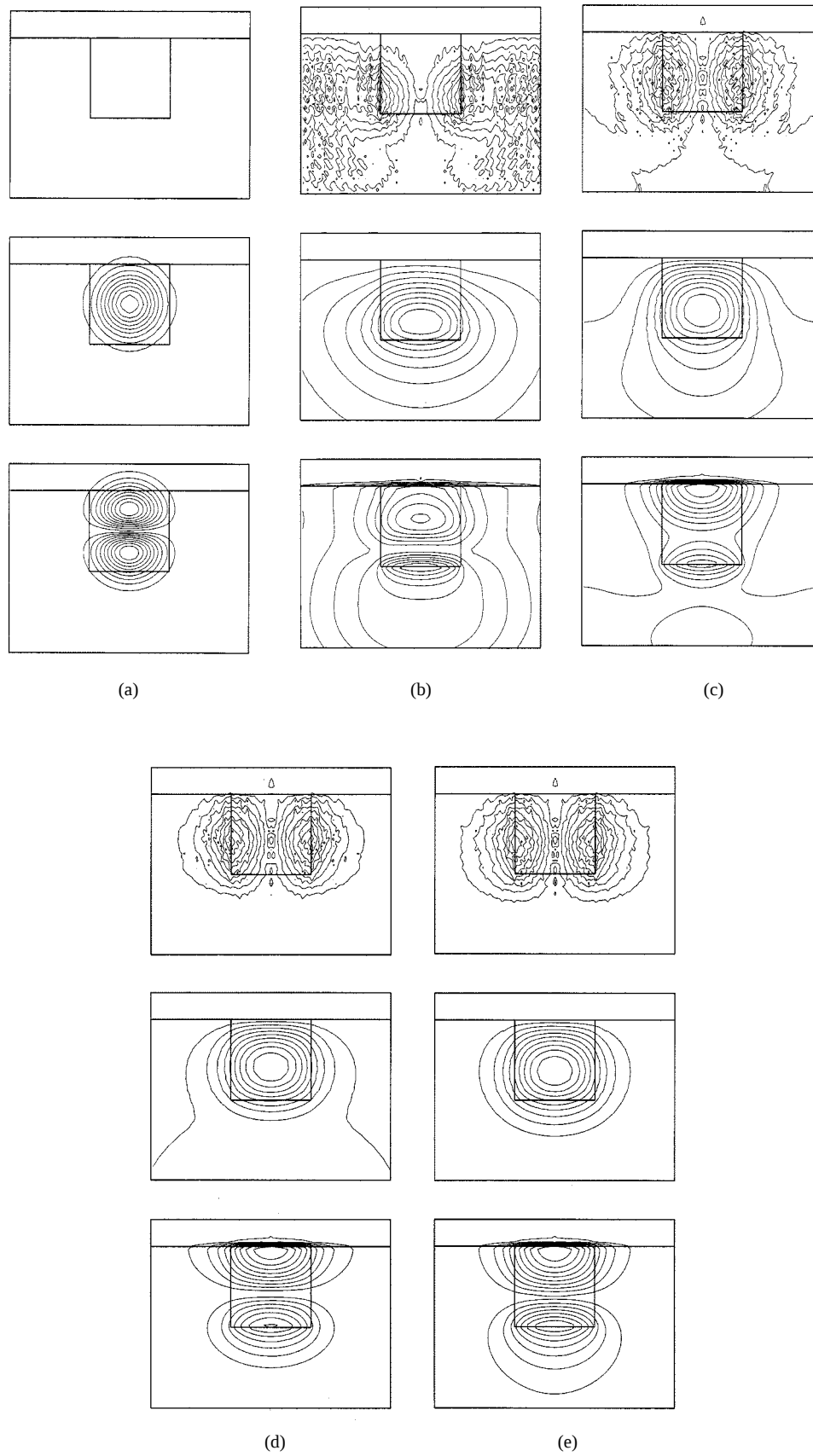


Fig. 6. (Top) E_x , (middle) E_y , and (bottom) E_z distributions at (a) $z = 0\mu\text{m}$, (b) $z = 40\mu\text{m}$, (c) $z = 80\mu\text{m}$, (d) $z = 120\mu\text{m}$, (e) $z = 160\mu\text{m}$, and (f) $z = 800\mu\text{m}$.

backward-difference scheme, is adopted here. In the BPM algorithm, backward reflections are neglected, so waveguide structures with abrupt refractive index changes along the z -direction should be carefully treated.

In order to improve numerical accuracy, the reference refractive index is renewed at each propagation step as follows:

$$n_{0,i} = \frac{1}{k_0} \text{Re} \left[\frac{-jb_i + \sqrt{-b_i^2 + 4a_i c_i}}{2a_i} \right] \quad (46)$$

with

$$a_i = \{\phi\}_i^\dagger [M]_i \{\phi\}_i \quad (47)$$

$$b_i = \{\phi\}_i^\dagger [L]_i \{\phi\}_i \quad (48)$$

$$c_i = \{\phi\}_i^\dagger [K]_i \{\phi\}_i \quad (49)$$

where † denotes complex conjugate and transpose.

V. NUMERICAL RESULTS

We consider a 3-D anisotropic optical waveguide surrounded by PMLs as shown in Fig. 4, where the substrate is LiNbO₃ (LN) and the guiding region is proton-exchanged LiNbO₃ (PE-LN). For simplicity, the thicknesses and the maximum values of α_j in (8), d_j , and $\alpha_{j,\max}$, are assumed to be the same in all the PML regions, and are denoted by d and $\alpha_{\max}$, respectively. The waveguide parameters are $t = w = 3 \mu\text{m}$, $W_x = 9 \mu\text{m}$, $W_y = 7 \mu\text{m}$, and $d = 2 \mu\text{m}$. The substrate has ordinary and extraordinary indexes of $n_o = 2.250$ and $n_e = 2.172$, and the refractive index difference between the guiding region and the substrate is $\Delta n_e = 0.01$ for the extraordinary index and for the ordinary index, Δn_o is assumed to be zero. The components of the symmetric relative permittivity tensor for LN are

$$\varepsilon_{xx} = n_o^2 \quad (50)$$

$$\varepsilon_{yy} = n_e^2 \cos^2 \theta_c + n_o^2 \sin^2 \theta_c \quad (51)$$

$$\varepsilon_{zz} = n_o^2 \cos^2 \theta_c + n_e^2 \sin^2 \theta_c \quad (52)$$

$$\varepsilon_{yz} = \varepsilon_{zy} = (n_o^2 - n_e^2) \cos \theta_c \sin \theta_c \quad (53)$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{zx} = \varepsilon_{xz} = 0 \quad (54)$$

where θ_c is the angle between the crystalline c -axis and the y -axis on the yz -plane. We subdivide one-half of the waveguide cross section into hybrid edge/nodal elements as shown in Fig. 5. The PML region is uniformly divided into 8 elements along the direction normal to the PML surface. The total number of elements are, respectively, 432 and 928 in the CW and the PML regions. As an initial field, vertically (y) polarized Gaussian beam is inputted. The wavelength is $\lambda = 0.84 \mu\text{m}$ and the beam spot size is $0.8 \mu\text{m}$. Figs. 6 and 7 show, respectively, the electric-field evolution and the reference index variation along the propagation distance, where $\theta_c = 30^\circ$, the propagation step size $\Delta z = 0.5 \mu\text{m}$, and the value of $\alpha_{\max}$ is taken as 8. No reflection from the CW edges and no instability have been observed, the radiation waves disappear out of the CW with increasing propagation distance, and the remaining field approaches that of the

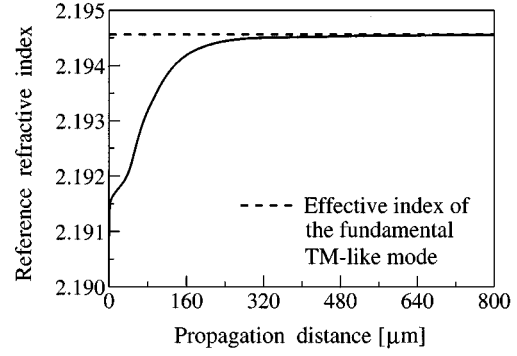


Fig. 7. Reference index variation along the propagation distance.

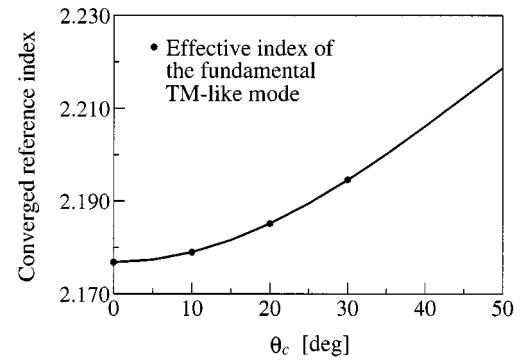


Fig. 8. Converged reference index as a function of crystal angle.

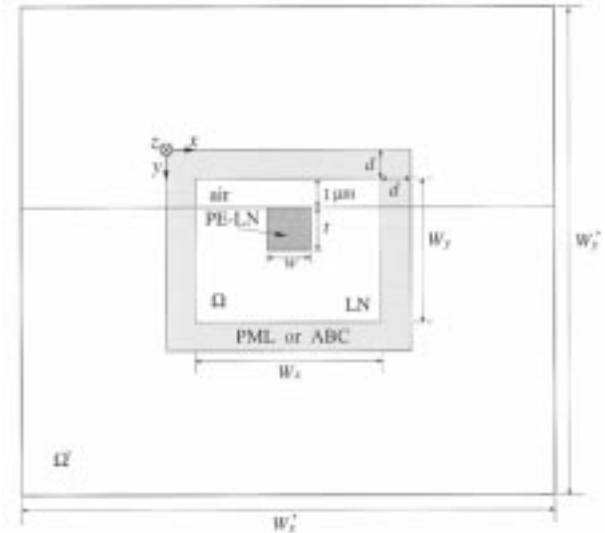


Fig. 9. Computational domains Ω and Ω' used in numerical experiments.

corresponding fundamental TM-like mode. The reference refractive index is also converged to the effective index of the fundamental TM-like mode which is obtained by the guided mode analysis. Fig. 8 shows the converged reference index as a function of crystal angle θ_c . The converged values of n_0 increase gradually with increasing θ_c , resulting in good agreement with

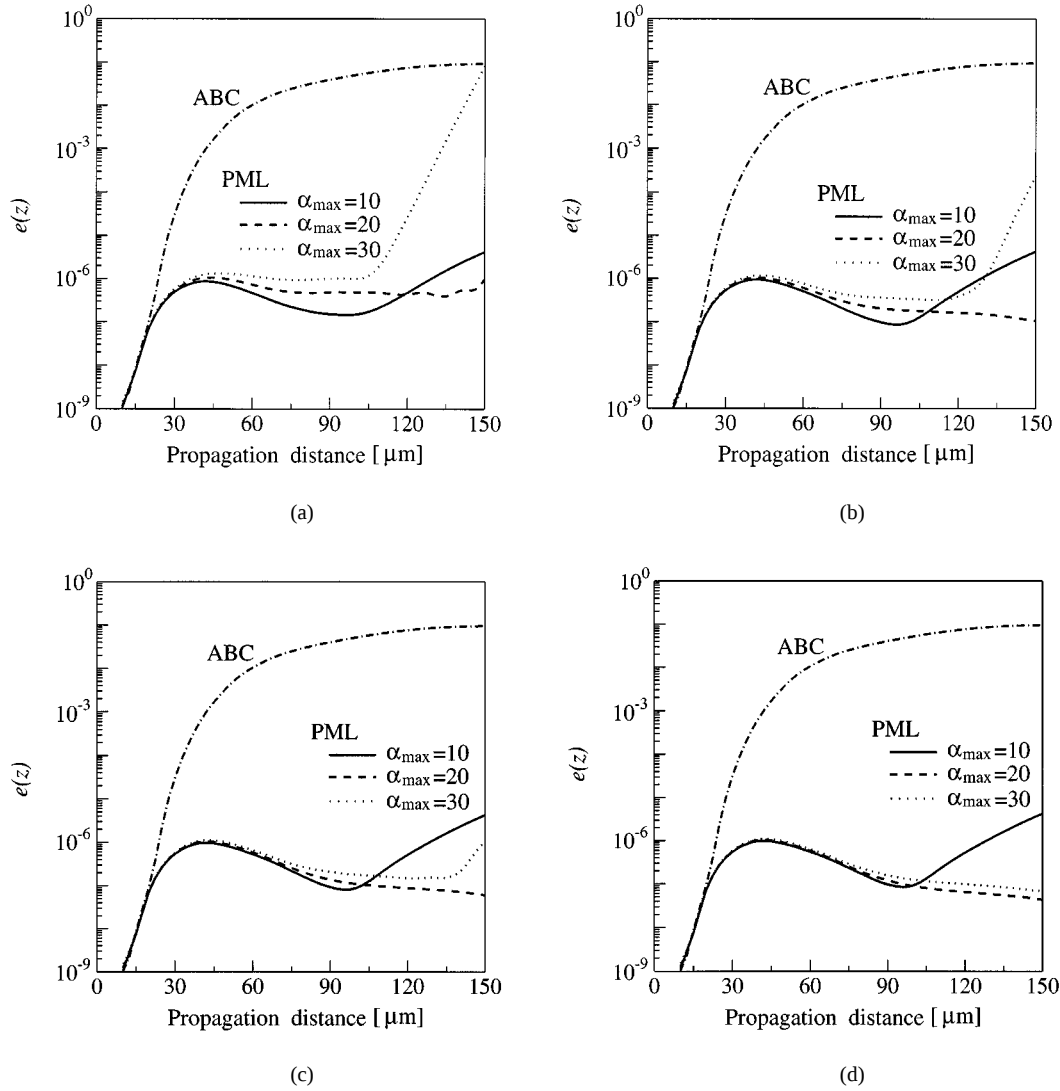


Fig. 10. Field-error variation along the propagation distance for $d = 1 \mu\text{m}$ and (a) $N = 6$, (b) $N = 8$, (c) $N = 10$, and (d) $N = 12$.

the effective indexes of the corresponding fundamental TM-like mode.

Next, in order to investigate numerical accuracy of the present PML, the field error along the propagation distance, $e(z)$, is defined as

$$e(z) = \frac{\iint_{\Omega} |\mathbf{E}(x, y, z) - \mathbf{E}'(x, y, z)|^2 dx dy}{\iint_{\Omega} |\mathbf{E}'(x, y, 0)|^2 dx dy} \quad (55)$$

where $\mathbf{E}$ is the electric field calculated by using computational domain Ω and $\mathbf{E}'$ is the one calculated by using larger domain Ω' as shown in Fig. 9, where the size of Ω is taken as $W_x = 9 \mu\text{m}$ and $W_y = 7 \mu\text{m}$ and that of Ω' is taken as $W'_x = 27 \mu\text{m}$ and $W'_y = 25 \mu\text{m}$. Here, the conventional ABC is also utilized, and the layer thickness d and the number of elements are the same as those in the PML case. In the ABC case, for each propagation

step, the optical field is multiplied by a window function $A(x, y)$ [7] as

$$A(x, y) = \begin{cases} \frac{1}{2} + \frac{1}{2} \cos \left[\pi \left(\frac{\rho}{d} \right)^3 \right], & \text{in ABC region} \\ 1, & \text{in non-ABC region} \end{cases} \quad (56)$$

where ρ is the distance from the beginning of ABC.

Figs. 10 and 11 show the error variation along the propagation distance for $d = 1 \mu\text{m}$ and $2 \mu\text{m}$, respectively, where vertically (y) polarized Gaussian beam is inputted, the inputted beam spotsize and wavelength are, respectively, $0.8 \mu\text{m}$ and $0.84 \mu\text{m}$, $\theta_c = 30^\circ$, $\Delta z = 0.5 \mu\text{m}$, and the value $\alpha_{\max}$ and the number of elements along the direction normal to the PML or the ABC surface, N , are taken as parameters. We can see that the PML with smaller values of $\alpha_{\max}$ cannot sufficiently absorb radiated waves even though increasing the value of N . The PML with larger values of $\alpha_{\max}$ reduces the numerical error with

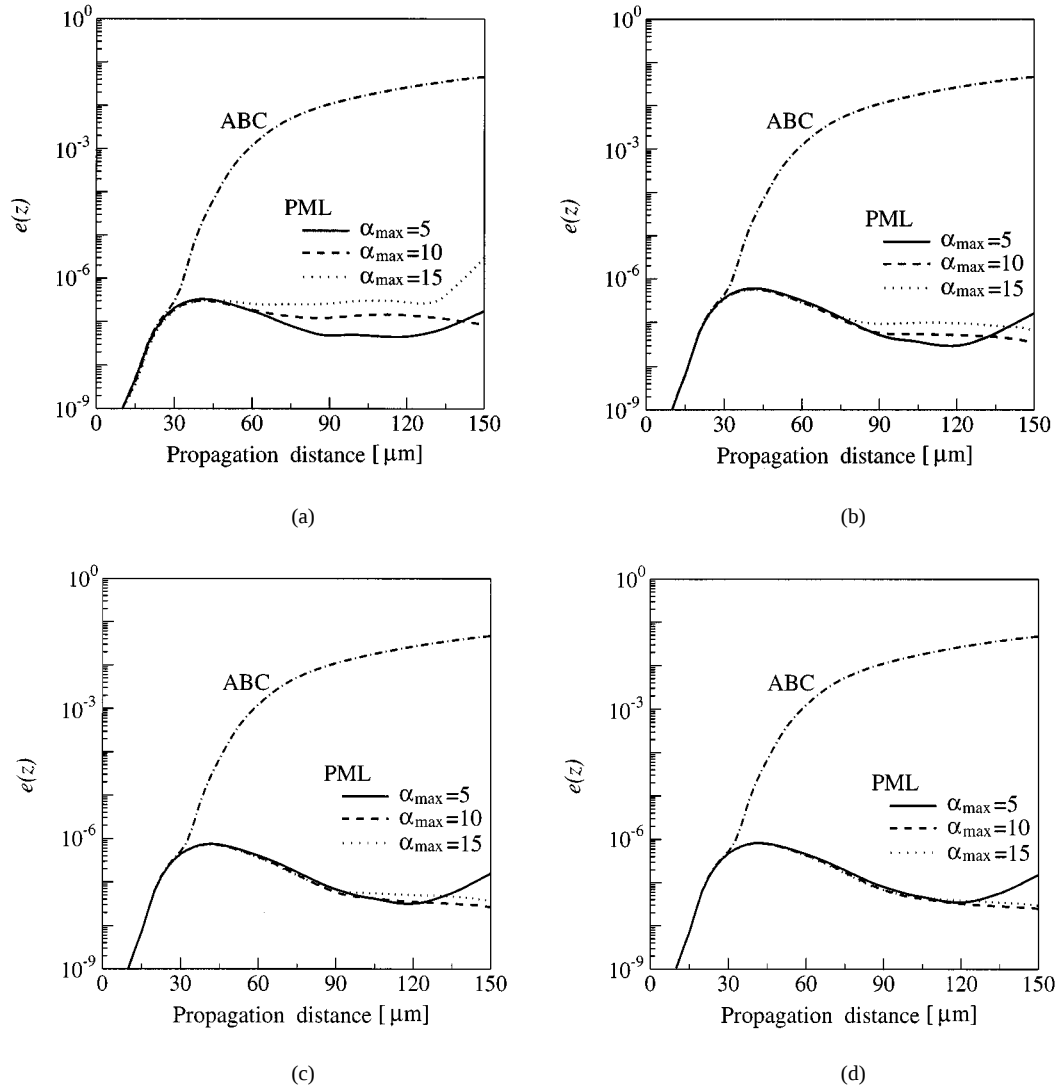


Fig. 11. Field-error variation along the propagation distance for $d = 2 \mu\text{m}$ and (a) $N = 6$, (b) $N = 8$, (c) $N = 10$, and (d) $N = 12$.

increasing N . All the PML results are superior to the conventional ABC ones. In order to keep the error lower than 10^{-6} , $\alpha_{\max} \geq 20$ and $N \geq 8$ are necessary for $d = 1 \mu\text{m}$, and $\alpha_{\max} \geq 10$ and $N \geq 8$ for $d = 2 \mu\text{m}$. We could say that when using PML, $2 \mu\text{m}$ is enough thickness to absorb radiated waves.

VI. CONCLUSION

The PML boundary condition for anisotropic media was, for the first time, incorporated into the VFE-BPM for the 3-D anisotropic optical waveguide analysis. To validate the present algorithm, numerical results for Gaussian beam propagation in proton-exchanged LiNbO₃ optical waveguides were presented. It is found that the present PML works well, compared with the conventional ABC.

REFERENCES

- [1] H.-P. Nolting and R. März, "Results of benchmark tests for different numerical BPM algorithms," *J. Lightwave Technol.*, vol. 13, pp. 216–224, Feb. 1995.
- [2] M. Koshiba and Y. Tsuji, "A wide-angle finite-element beam propagation method," *IEEE Photon. Technol. Lett.*, vol. 8, pp. 1208–1210, Sept. 1996.
- [3] Y. Tsuji, M. Koshiba, and T. Shiraishi, "Finite element beam propagation method for three-dimensional optical waveguide structures," *J. Lightwave Technol.*, vol. 15, pp. 1728–1734, Sept. 1997.
- [4] E. Montanari, S. Sellerei, L. Vincetti, and M. Zoboli, "Finite-element full-vectorial propagation analysis for three-dimensional z -varying optical waveguides," *J. Lightwave Technol.*, vol. 16, pp. 703–714, Apr. 1998.
- [5] D. Schulz, C. Gingener, M. Bludsuweit, and E. Voges, "Mixed finite element beam propagation method," *J. Lightwave Technol.*, vol. 16, pp. 1336–1341, July 1998.
- [6] J.-P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, pp. 185–200, Oct. 1994.
- [7] J. Sajionmaa and D. Yevick, "Beam-propagation analysis of loss in bent optical waveguides and fibers," *J. Opt. Soc. Amer.*, vol. 73, pp. 1785–1791, Dec. 1983.
- [8] K. T. Koai and P.-L. Liu, "Modeling of Ti:LiNbO₃ waveguide devices: Part II—S-shaped channel waveguide bends," *J. Lightwave Technol.*, vol. 7, pp. 1016–1021, July 1989.
- [9] G. R. Hadley, "Transparent boundary condition for the beam propagation method," *Opt. Lett.*, vol. 16, pp. 624–626, May 1991.
- [10] Ü. Pekel and R. Mittra, "A finite-element method frequency-domain application of the perfectly matched layer (PML) concept," *Microw. Opt. Technol.*, vol. 9, pp. 117–122, June 1995.

- [11] Z. S. Sacks, D. M. Kingsland, R. Lee, and J. F. Lee, "A perfectly matched anisotropic absorber for use as an absorbing boundary condition," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 1460–1463, Dec. 1995.
- [12] M. Koshiba, Y. Tsuji, and M. Hikari, "Finite-element beam propagation method with perfectly matched layer boundary conditions," *IEEE Trans. Magn.*, vol. 35, pp. 1482–1485, May 1999.
- [13] A. Cucinotta, G. Pelosi, S. Selleri, L. Vincetti, and M. Zoboli, "Perfectly matched anisotropic layers for optical waveguide analysis through the finite-element beam-propagation method," *Microw. Opt. Technol. Lett.*, vol. 23, pp. 67–69, Oct. 1999.
- [14] Y. Tsuji, M. Koshiba, and N. Takimoto, "Finite element beam propagation method for anisotropic optical waveguides," *J. Lightwave Technol.*, vol. 17, pp. 723–728, Apr. 1999.
- [15] F. L. Teixeira and W. C. Chew, "General closed-form PML constitutive tensors to match arbitrary bianisotropic and dispersive linear media," *IEEE Microwave Guided Wave Lett.*, vol. 8, pp. 223–225, June 1998.
- [16] J.-F. Lee, D.-K. Sun, and Z. J. Cendes, "Full-wave analysis of dielectric waveguides using tangential vector finite element," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1262–1271, Aug. 1991.
- [17] A. F. Peterson, "Vector finite element formulation for scattering from two-dimensional heterogeneous bodies," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 357–365, Mar. 1994.
- [18] M. Koshiba and Y. Tsuji, "Curvilinear hybrid edge/nodal elements with triangular shape for guided-wave problems," *J. Lightwave Technol.*, vol. 18, pp. 737–743, May 2000.



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